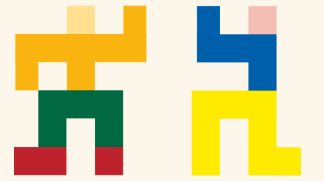


XORTING



Statement:

There is a hidden permutation p of size N . You can ask queries to obtain the value of $p[i] \wedge p[j]$. Your task is to find the lexicographically minimal permutation that produces the same pairwise XOR values as the hidden one.

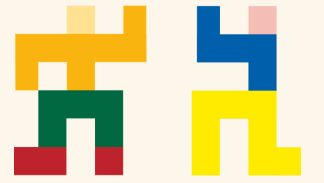
Constraints:

$$N \leq 2^{20}$$

$$Q \leq N \text{ (number of queries)}$$



XORTING



Initial observations:

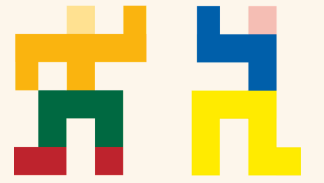
- 1) $N-1$ queries give us all the information: $\text{query}(x,y) = \text{query}(0,x) \oplus \text{query}(0,y)$.
- 2) $p[0]$ defines the permutation: $p[x] = p[0] \oplus \text{query}(0,x)$.
 - 2.1) Lexicographic minimum $\Leftrightarrow p[0]$ is minimal.
- 3) The values of $\text{query}(0,x)$ are pairwise distinct.
 - 3.1) The values of $(v \oplus \text{query}(0,x))$ are mutually different.
 - 3.2) $p[0] = v$ is valid $\Leftrightarrow \max (v \oplus \text{query}(0,x)) = n - 1$.

$O(N^2)$ **Solution:**

Implement N linear checks for 3.2.

Off-syllabus: Implement this with a Trie for $O(N \log N)$, giving $O(N)$ amortised with caching query information.

XORTING



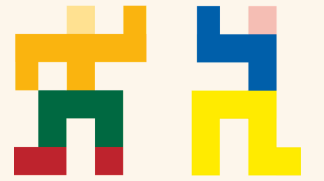
N odd solution:

Define $\text{cnt}[b][v]$ to be the number of $\text{query}(0,x)$ having bit v at position b
 N odd $\Rightarrow \text{cnt}[b][0] \neq \text{cnt}[b][1] \Rightarrow$ if $\text{cnt}[b][0] > \text{cnt}[b][1]$, $p[0]$ has bit 0 at position b , otherwise bit 1. Reconstruct the unique possible permutation.

Full solution:

- The same approach, but when $\text{cnt}[b][0] = \text{cnt}[b][1]$, it can be proven that there exists a solution in which $p[0]$ has bit 0 at every such position.
- Pick bits greedily for $p[0]$ so as to minimize it.
- Naive implementation for $O(N \log N)$ would likely pass (small constant).
- can be optimised to $O(N)$ with bitwise operations.

XORTING



Summary:

Points by complexity:

- $O(N^2)$: 40 points
- $O(N \log N)$: 75 (100) points
- $O(N)$: 100 points

Mentioned approaches:

- Elegant ad hoc bit counting
- *Off-syllabus*: Trie

Other approaches, mimicking the ideas (In editorial):

- *On-syllabus*: Prefix-sum maintenance of the trie
- Other bit-pattern analysis solutions