AUTOMATA

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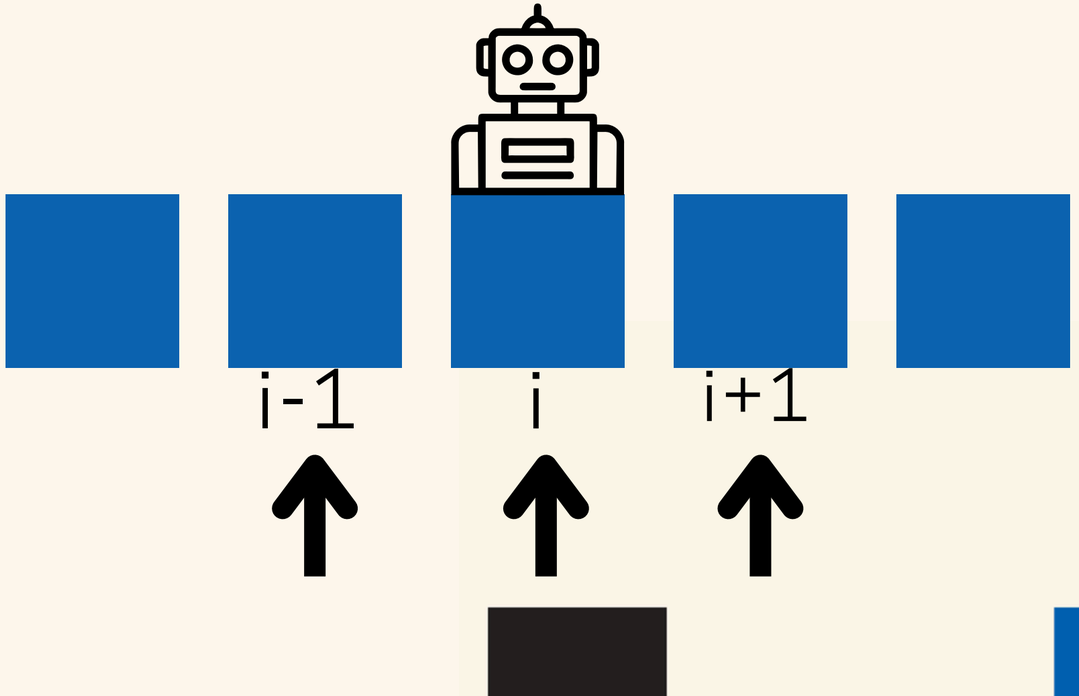

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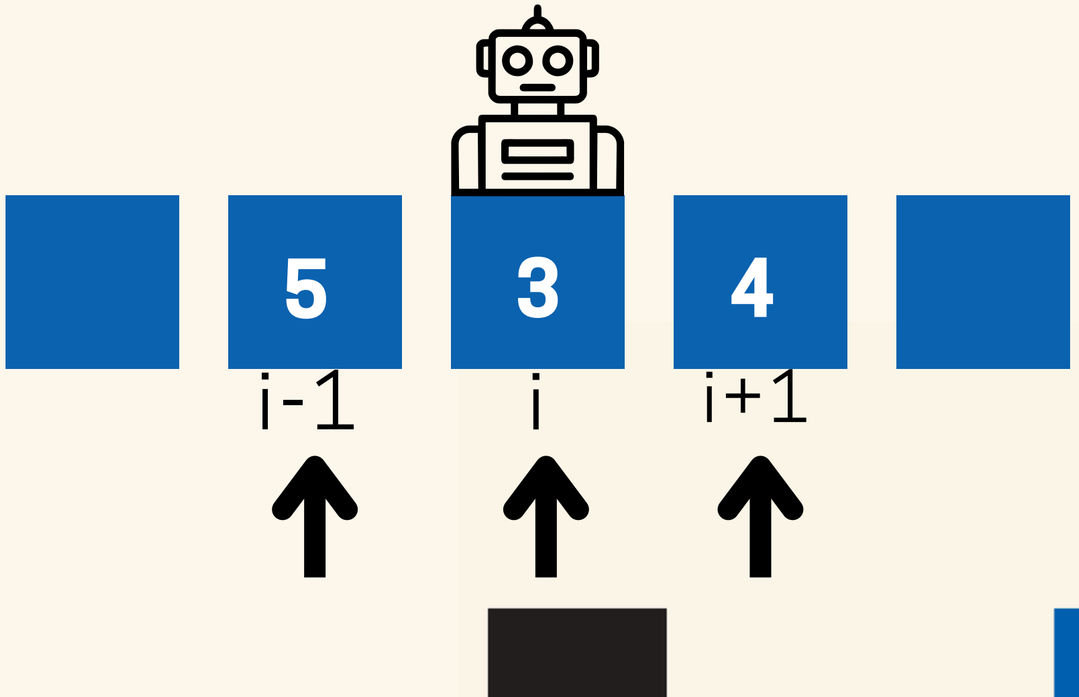

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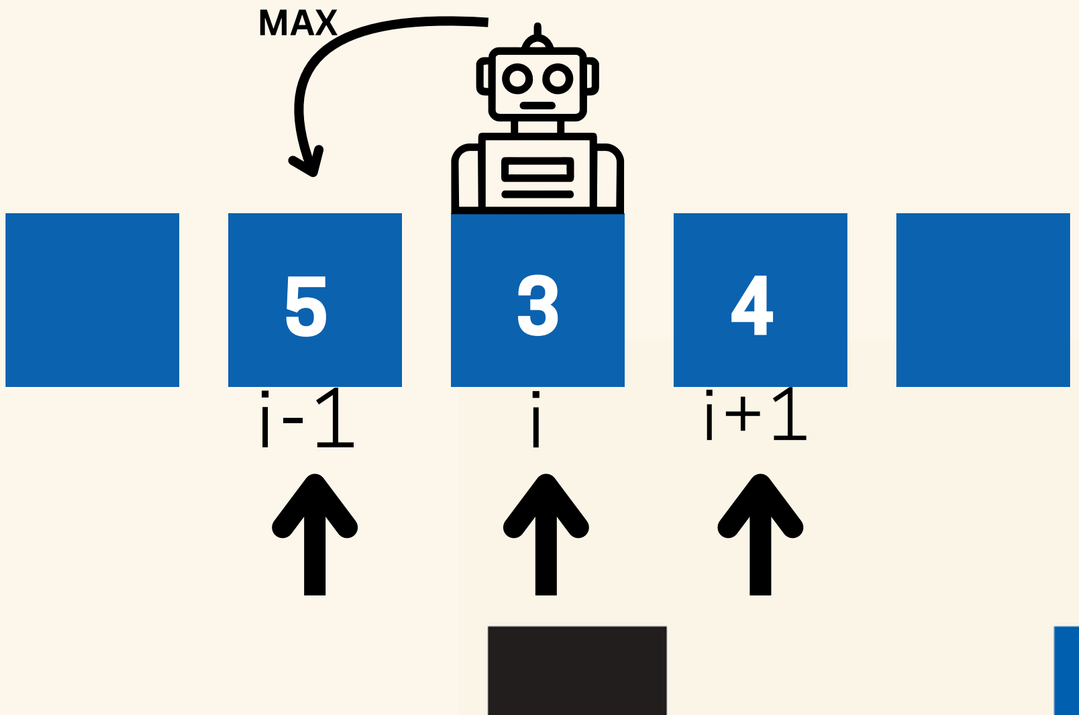

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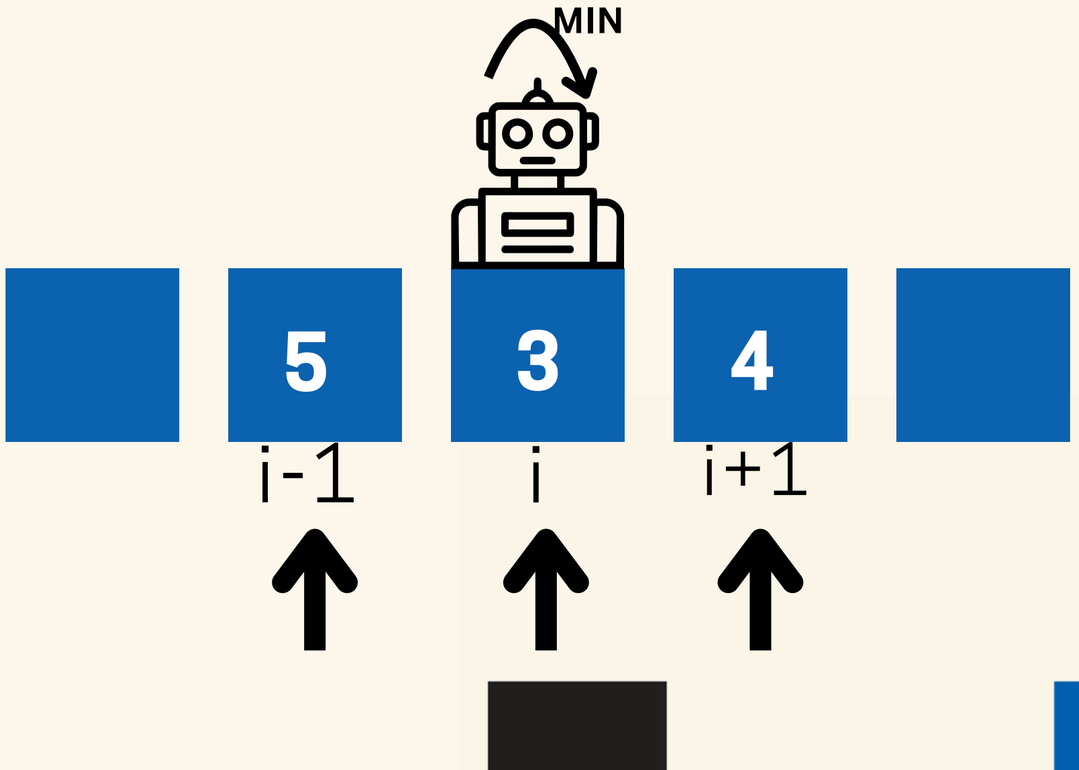


STATEMENT

TASK:

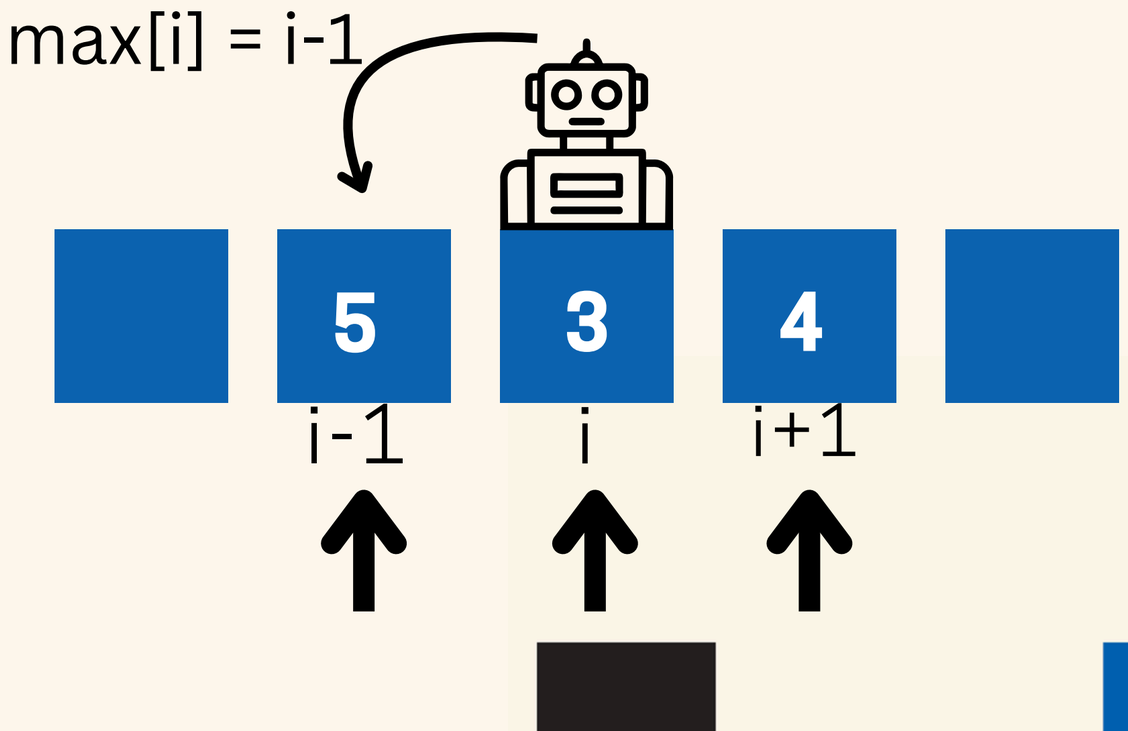
- Given Q queries
- Each query gives positions X[0], ..., X[K-1]
- If we place robots there, *does some program exist,* s.t. the robots finish at the same cell

CONSTRAINTS

- N <= 2*10^5
- Q <= 5*10^5
- The sum of K over all queries does not exceed 10^6
- The queries must be answered online (the moment they are given)

NOTATION

- Two helper arrays: max[i], min[i]



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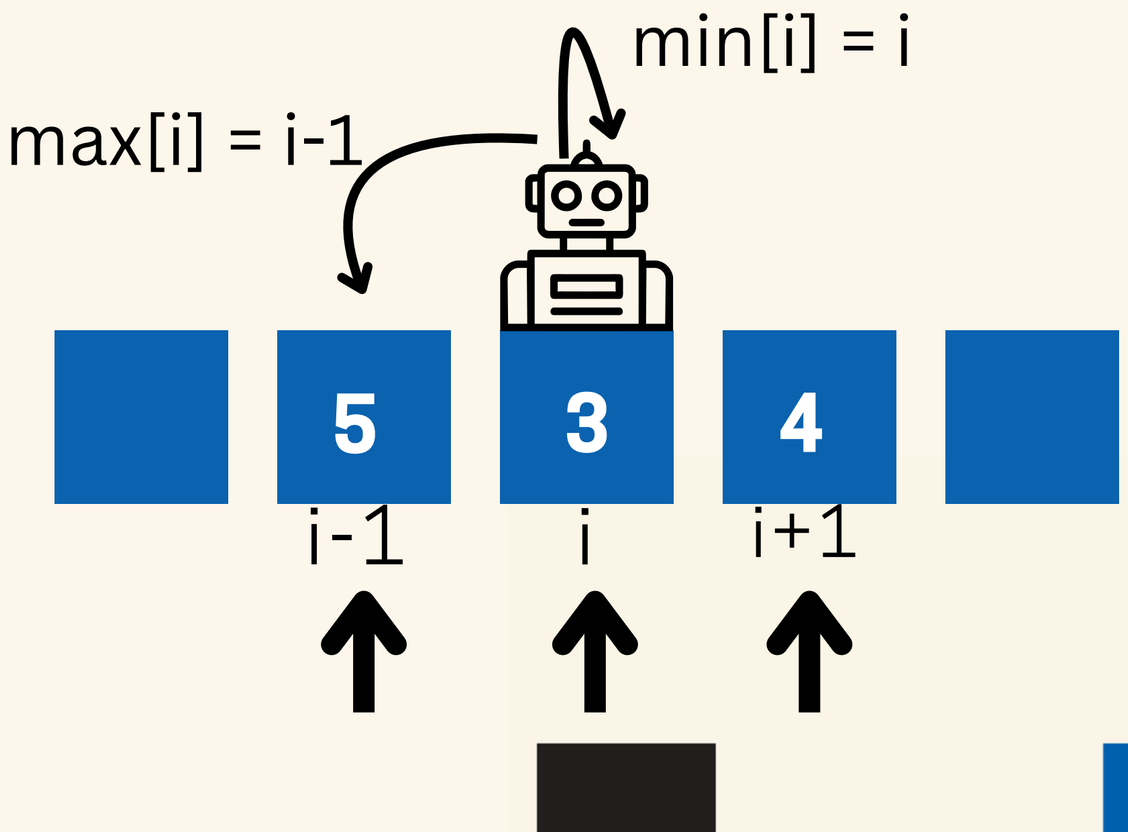

NOTATION

- Two helper arrays: max[i], min[i]

OBSERVE:

- **after getting MAX command, you go i->max[i]**
- **similarly for MIN, you go i->min[i]**

SUBTASK 1

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Two options:

- [MIN] → works if min[x0] == min[x1]
- [MAX] → works if max[x0] == max[x1]

SUBTASK 2

CONSTRAINTS:

- **K=2**
- **if a program exists, it is at most 5 commands**

Idea: try all possible 5-command combinations

SUBTASK 1-5

MAIN CONSTRAINT: K=2

- IDEA: Think about the given query with two positions L and R as graph problem.
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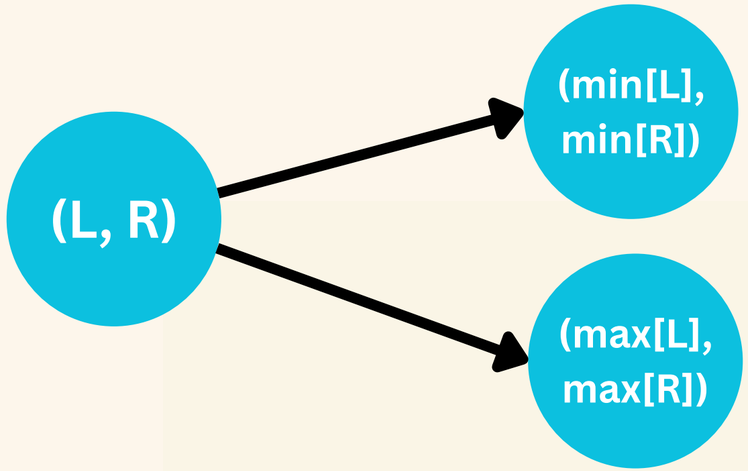

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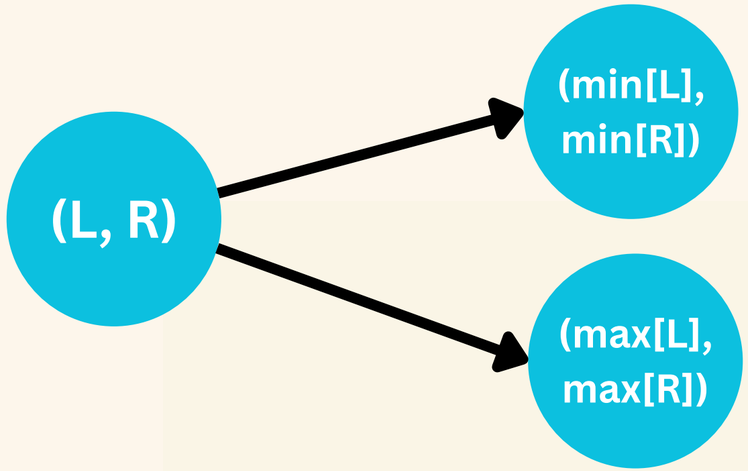
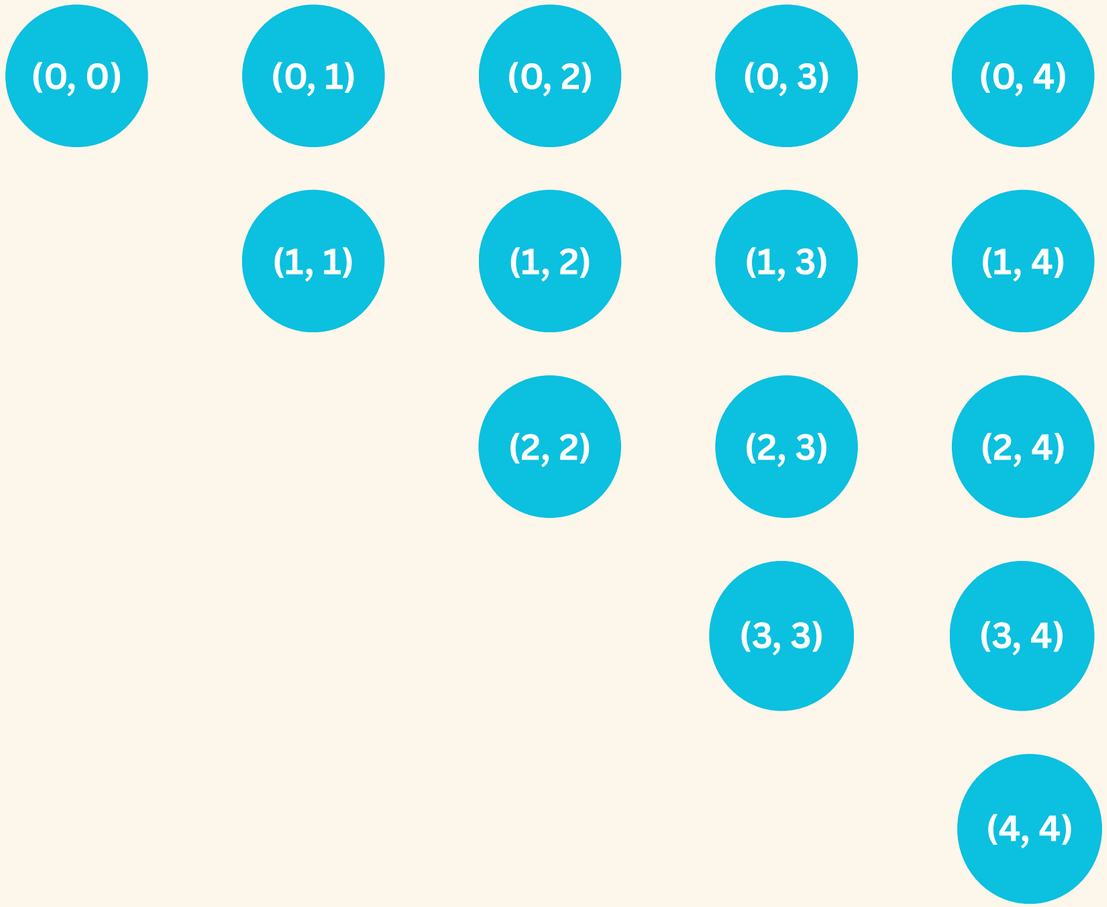


GOAL: From (L, R) move to some position (i, i)

EXAMPLE

P = [3, 2, 1, 0, 4]

1. **STATES**



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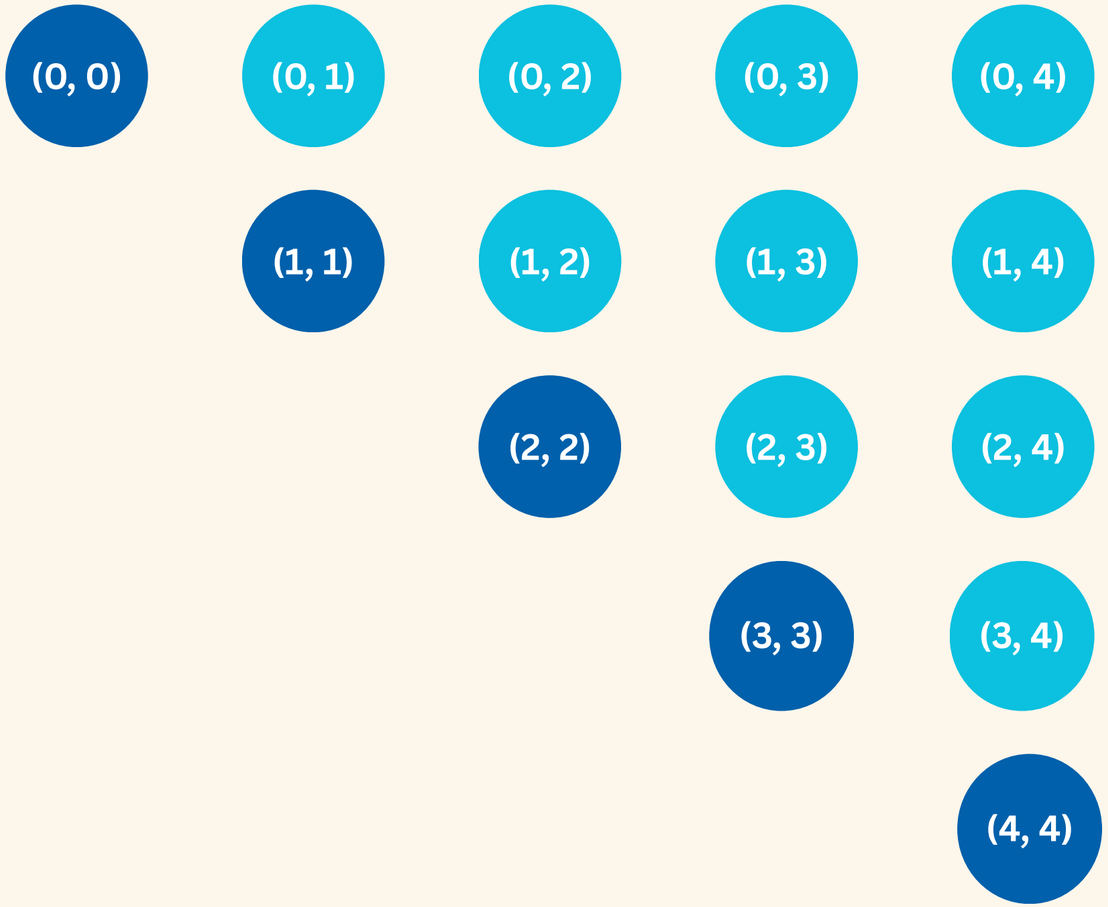
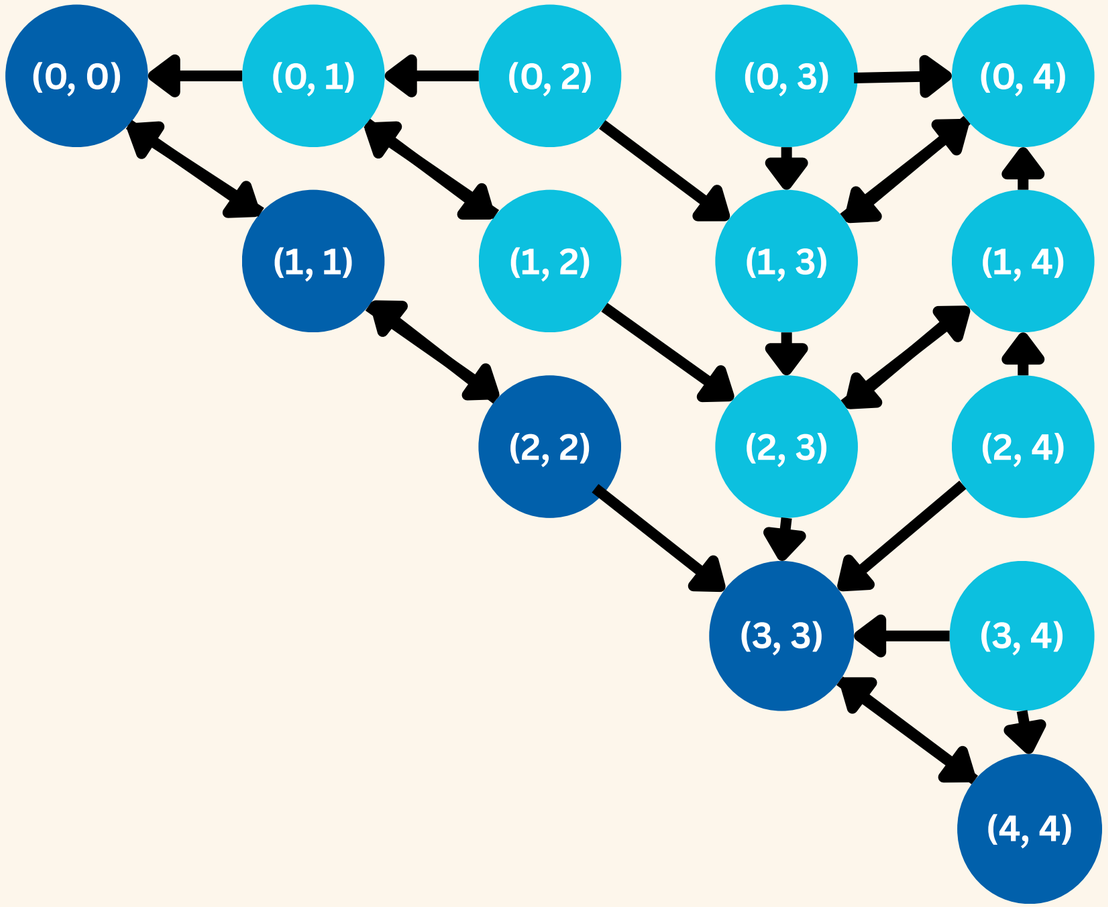


EXAMPLE

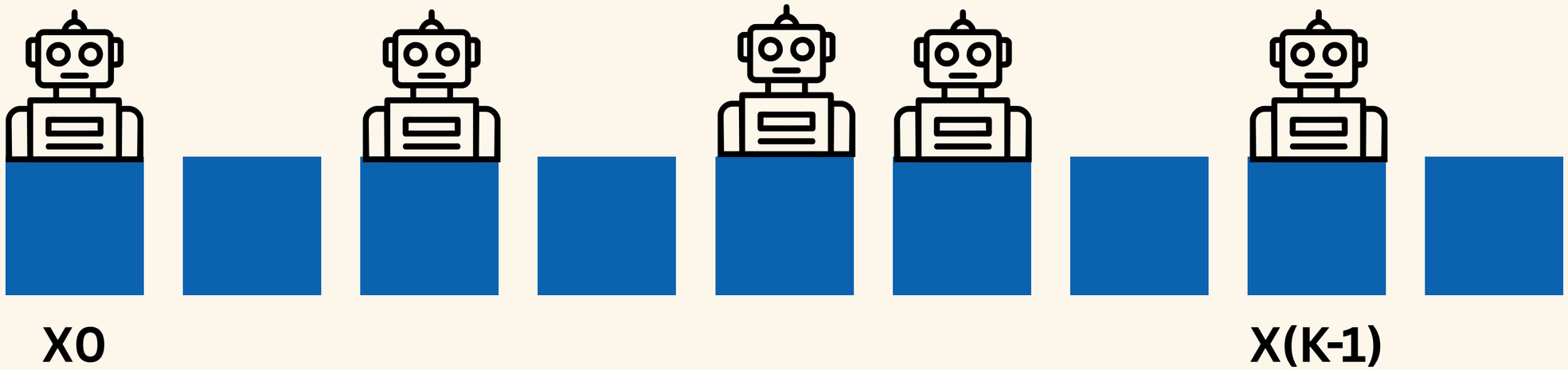
P = [3, 2, 1, 0, 4]

1. **STATES**
2. **SPECIAL STATES**
3. **EDGES**



OBSERVATION

We only care about the left-most and the right-most possible positions: at X[0] and X[K-1].



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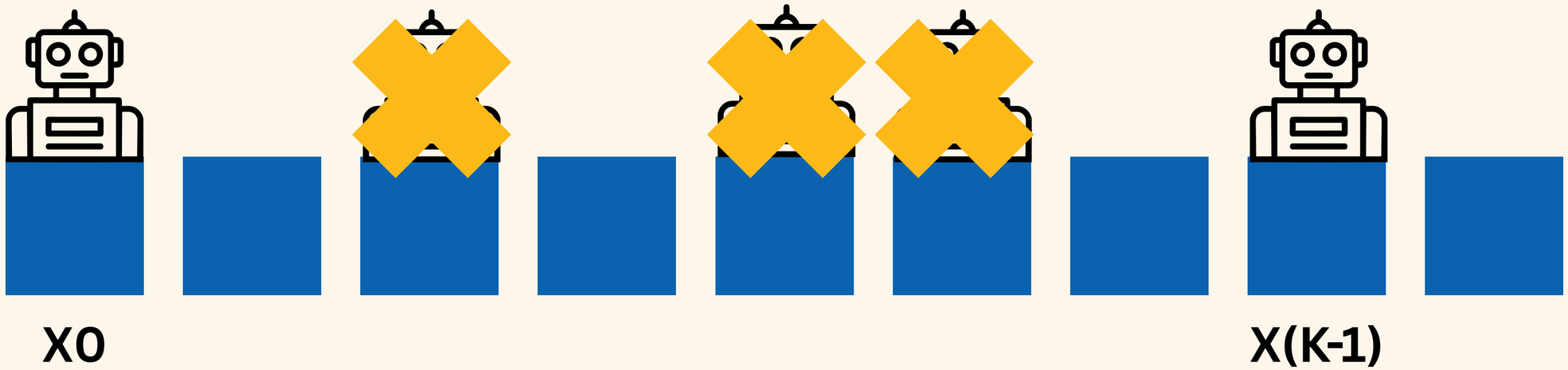

OBSERVATION

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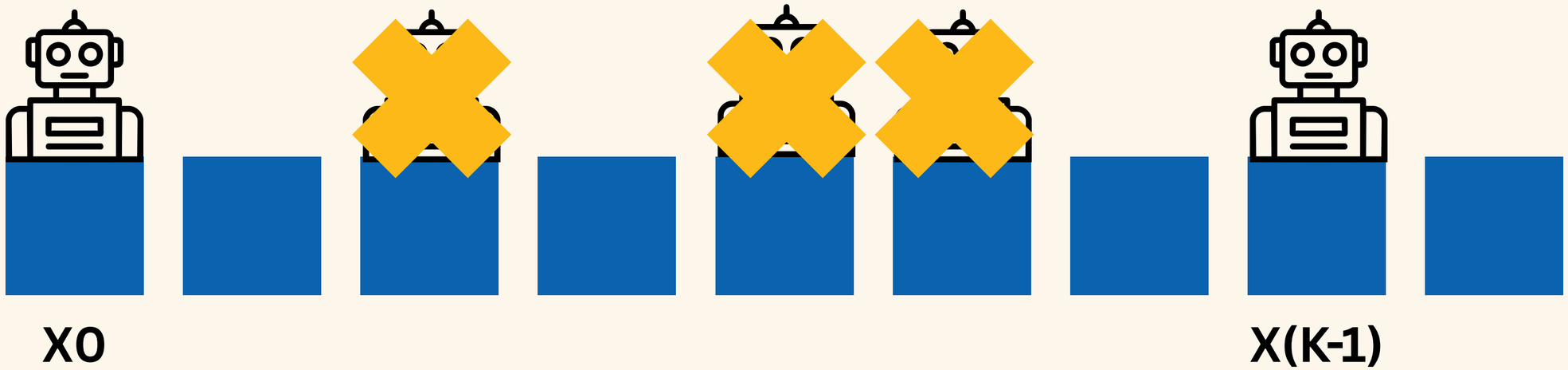
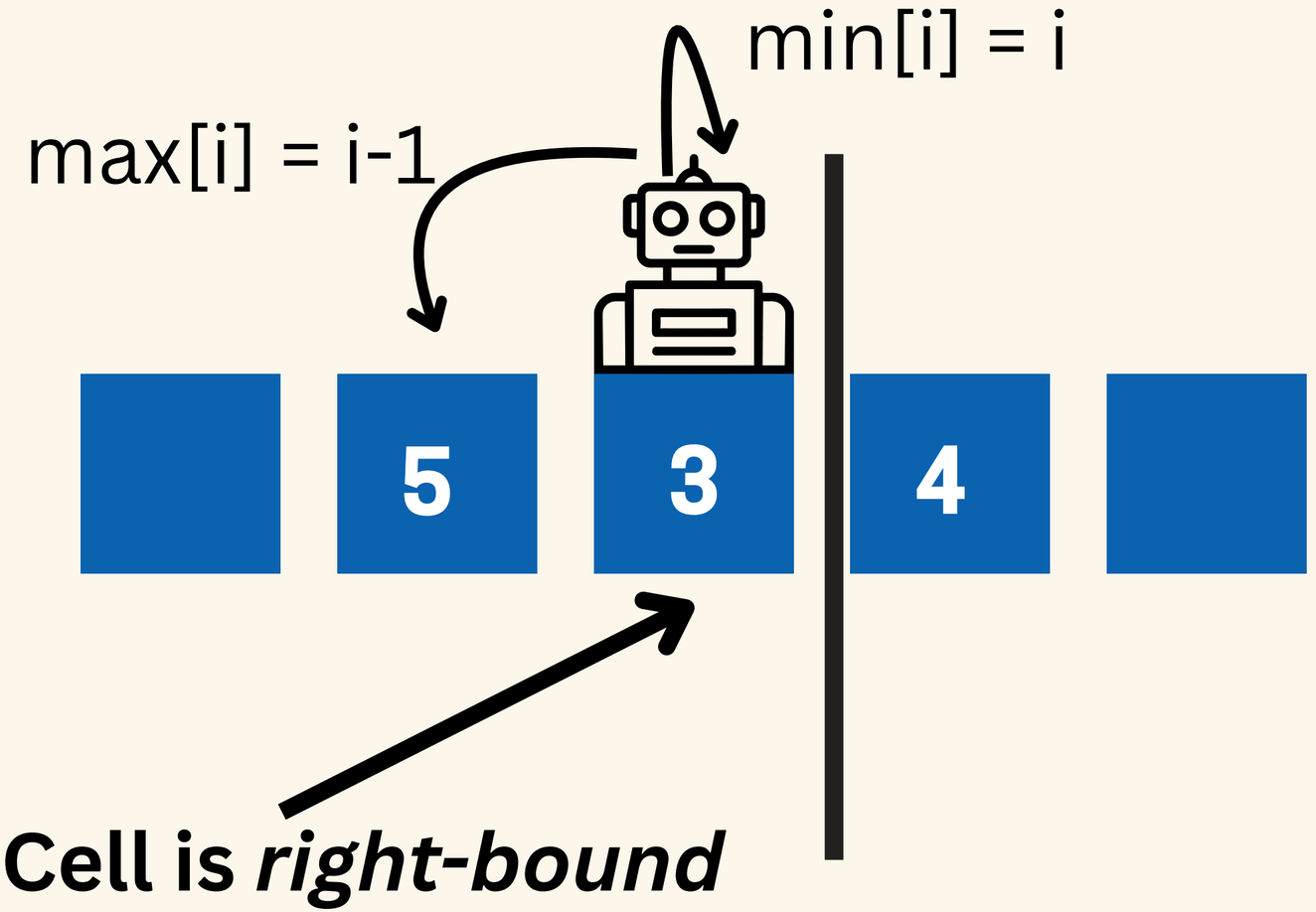


BOUNDS



BOUNDS

- **i** is *left-bound:* max[i] >= i AND min[i] >= i.
- **i** is *right-bound*: max[i] <= i AND min[i] <= i.

SOLUTION

We need to answer query X, Y (two robot positions), and say if it is *good*.

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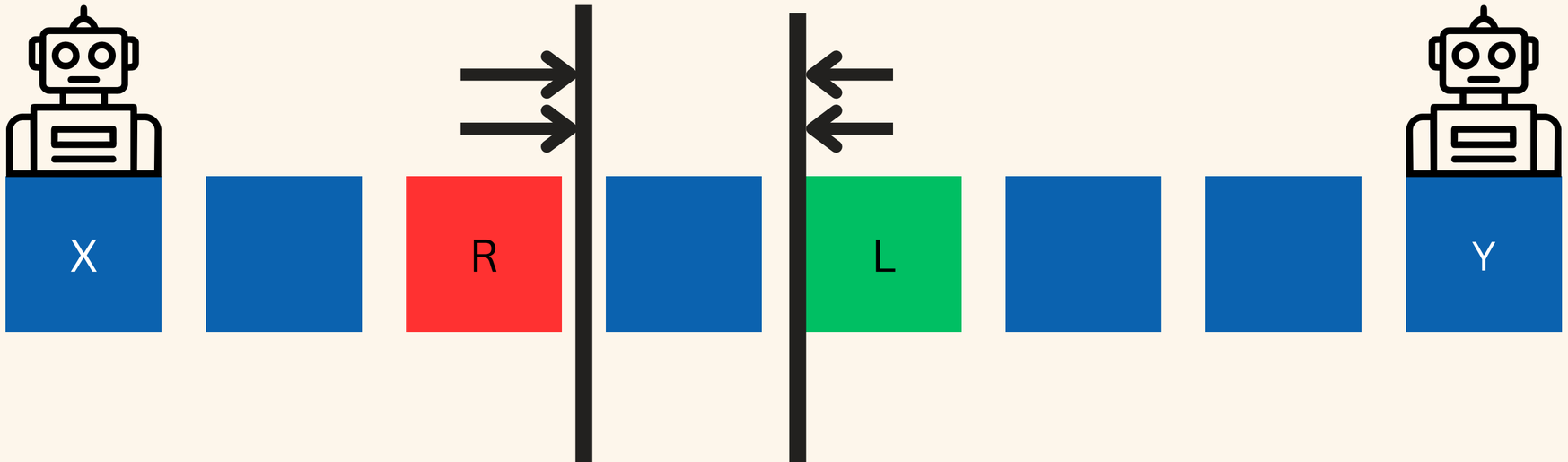
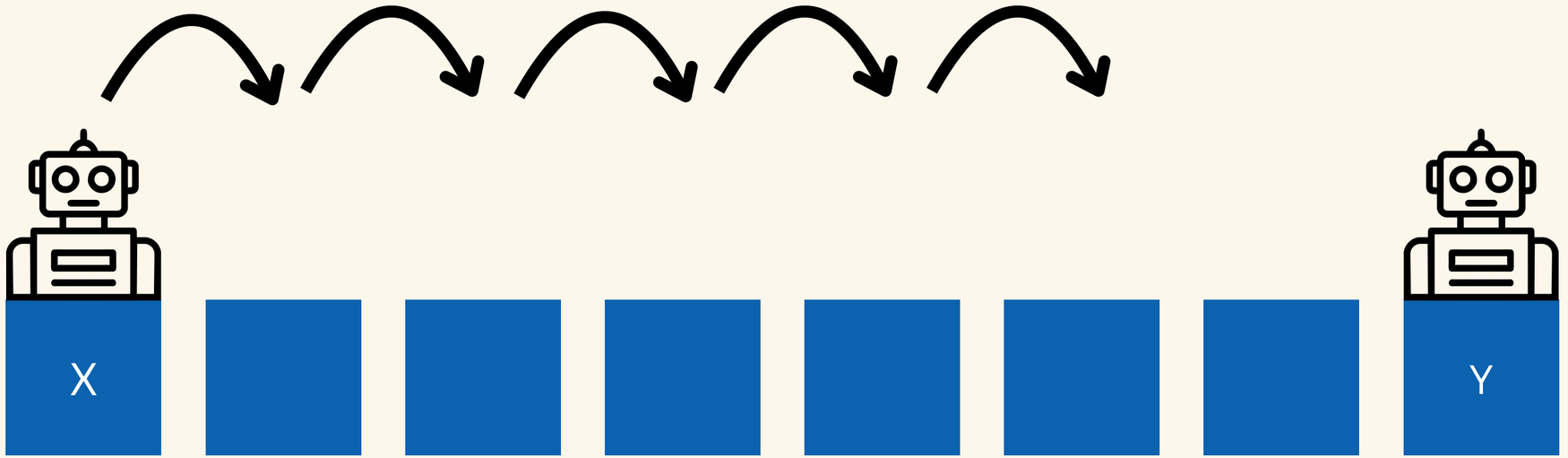
- Case X==Y → good
- Case when there is X <= R < L <= Y → bad



SOLUTION

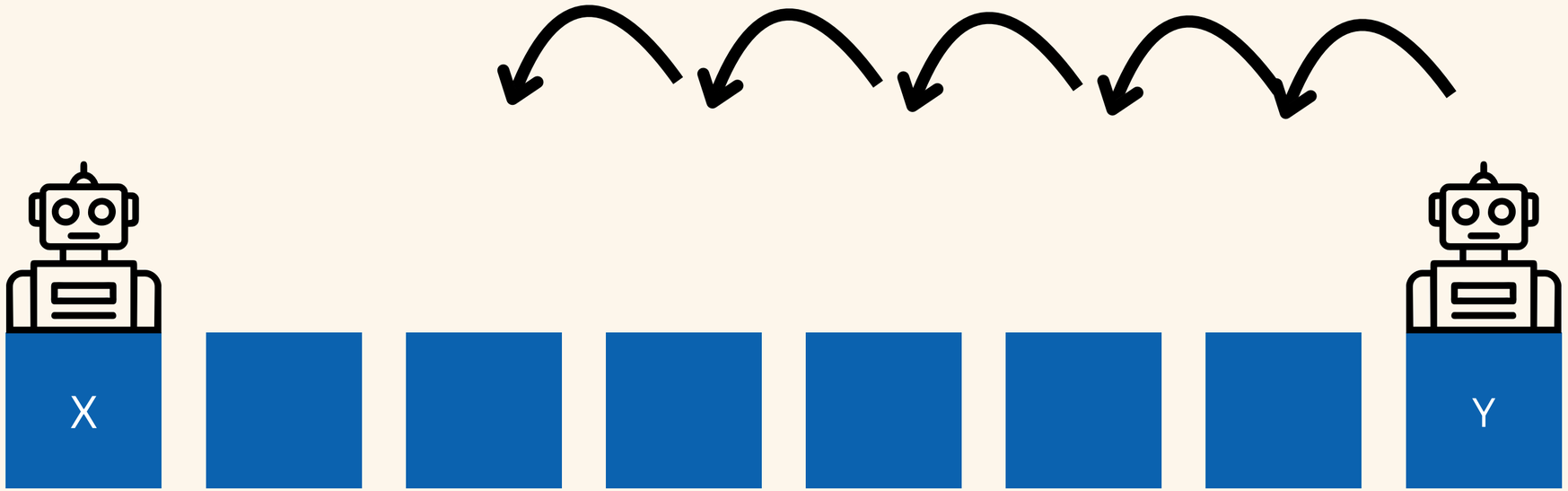
Case: no right-bound in range [X, Y)

Idea: robot from X can be moving to the right until it meets the other one



SOLUTION

Same idea for when there is no left-bound in (X, Y].



SOLUTION

Hardest case: between X and Y all left-bounds are to the left of all right-bounds.

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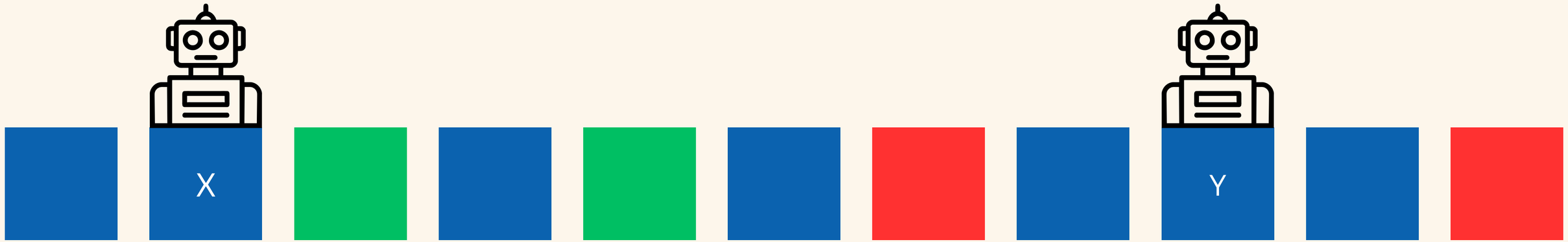
If the **first** bound to the right of Y is **right-bound** → good
(or, if first to the left of X is left-bound → good)



SOLUTION

Otherwise, check if X can reach a good bound before Y reaches a bad bound.